MATHEMATICAL MODELING AND GLOBAL PROCESSES

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Summary

At present, the increasing role of mathematics in the life of society and in the development of science and technology is remarkable. A mathematician, as earlier an engineer, becomes a necessary participant of industrial process and social life. Mathematics is more and more widely employed in various spheres of human activity. Fast complication of diverse activities in the society conditioned by a high level of productive forces and deep specialization of the production is one of the main causes of rapid expansion of mathematical methods (including mathematical modeling and the theory of optimal control).

The requirements of rational and efficient use of natural resources (which are far from limitless on our planet) and the necessity to comprehend close and remote ecological consequences of human industrial activity are another causes of the mathematization process.

The application of the methods of mathematical modeling and control theory on the basis of wide use of modern computer facilities is one of the most important tools in increasing the quality of decision-making in the modern society and industrial activity of humanity on this planet.

When making a decision with the use of computer technologies, the managers and researchers want to know, if all possible interesting variants were considered? What ideas underlie the estimation of consequences of possible variants of solution? How to formulate the indicators characterizing the efficiency of a system about which the decisions are made? How to select the most appropriate solution? To answer these questions, a problem to be analyzed must be described exactly. Mathematics is the language most appropriate for this purpose.

The application of mathematical methods in society was always associated with economics at all stages of social development. The concept of economics as a science originated in the flourishing period of the Greek slave-owning democracy, when the

first attempts were made not only to simply note, but also theoretically comprehend the facts of economic life.

The development of practical methods of rational management, rational planning, and rational control of the activities in society and in any state involves planning and coordination of demands and resources on the state scale, the coordination of adjacent industrial branches, and the determination of proportions between different branches within the framework of an individual state or several states. These questions become more and more urgent at the present stage of human development, when we observe the globalization processes in many spheres.

The issues noted above and a number of other global problems always exerted a great influence on the life of society on this planet. In examining and solving these problems, the methods of mathematical modeling, various mathematical models, and the theory of optimal control play an important role.

1. Introduction

In modern life, very important processes occur, generated by the intense development of science and technology; consequently, over two or three decades, changes in the life of people and in their world outlook happen such that would have taken centuries in the past. The increasing role of mathematics in the life of society and in the development of science and technology is one of these processes. A mathematician, as earlier an engineer, becomes a necessary participant in industrial process and social life. Mathematics is more and more widely employed in various spheres of human activity. Fast complication of diverse activities in society conditioned by the high level of productive forces and deep specialization of production is one of the main causes of the rapid expansion of mathematical methods (including mathematical modeling and the theory of optimal control). The requirements of rational and efficient use of natural resources (which are far from limitless on our planet) and the necessity to comprehend the close and the remote ecological consequences of human industrial activity are another cause for the mathematization process. The application of the methods of mathematical modeling and control theory on the basis of the wide use of modern computer facilities is one of the most important tools in increasing the quality of decision-making in modern society and the industrial activity of humanity on this planet.

Industrial activity in society was always associated with arithmetical or geometrical calculations. More than that, mathematics itself originated from practical and industrial needs. The advent of electronic computational technology, which, at first glance, did not differ essentially from conventional arithmometers (though, much faster): just traditional calculations, taking many hours and days, now were performed in seconds. Thus, the estimation of many variants instead of only one became possible. The results of these calculations, presented to the person responsible for decision-making, provide him/her a possibility to choose the best variant among all considered. When making a decision with the use of computer technologies, the managers and researchers want to know, if all possible interesting variants were considered? What ideas underlie the estimation of the consequences of possible variants of solution? How to formulate the indicators characterizing the efficiency of a system about which the decisions are made? How to select the most appropriate solution?

To answer these questions, a problem to be analyzed must be described exactly. Mathematics is the language most appropriate for this purpose. The description of a system under investigation leads to mathematical model of the latter. In addition to the descriptive tools, mathematics provides an apparatus for analyzing the model, which allows the investigation of its properties and the selection of the most suitable solution. Thus, the implementation of computer technology, as a system for processing the information, into everyday practice leads to a principally new stage: to the construction of mathematical models of a vast variety of processes and to their analysis.

However, there is another side in the penetration of mathematical methods into various investigations. The application of mathematical methods in the sciences means further development, moreover, accelerated development, which ensures better comprehension of the processes that occur in human society. Being based on the high level of sciences, deep understanding of processes in society, and on the ability to use this understanding in practice, the construction of mathematical models can significantly improve the system of planning and controlling these processes.

The application of mathematical methods in society was always associated with economics at all stages of social development. The concept of economics as a science originated in the flourishing period of the Greek slave-owning democracy, when the first attempts were made not only to simply note, but also theoretically comprehend the facts of economic life.

The word "economy," which is the origin of "economics," "economical science," etc., being translated from Greek, means the science on housekeeping. By its major content, it was to deal with the questions of rational management. However, in modern terminology, the term "rational management" can be replaced by the term "optimal control."

The development of practical methods of rational management, rational planning, and rational control of the activities in society and in any state involves planning and the coordination of demands and resources on the state scale, the coordination of adjacent industrial branches, and the determination of proportions between different branches within the framework of an individual state or several states. These questions become more and more urgent at the present stage of human development, when we observe the globalization processes in many spheres.

A new stage in the development of the methods of mathematical, economical-mathematical modeling and the theory of optimal control started in the late fifties of the twentieth century, when the advent of computing facilities made the multi-variant plan calculations on the basis of economical-mathematical models realizable, at least principally. The development of economical and mathematical methods at that time was significantly influenced by the works of Kantorovich and Tauns, who, as a result of an analysis of some problems on planning production, had formulated a new important class of mathematical problems, called the problems of linear programming. In linear programming, they search, among all permissible solutions satisfying a system of linear equations and inequalities, for a solution providing a maximum (or minimum) to certain linear criteria. At present, linear programming is the major mathematical method for analyzing problems of production planning.

Starting in the early sixties, the choice of optimal solution based on a numerical (with the use of computers) analysis of economical-mathematical models intensively penetrated all divisions of economics. At that time, it was proposed to use an optimization approach in planning the people's industry and in solving such important problems as the construction of a price system and so on. These years witnessed fast development of some divisions of mathematics associated with the solution of optimization problems: linear and nonlinear programming, the theory of optimal control, dynamic programming, etc.

In the early seventies, mathematical modeling became an acknowledged tool in analyzing economical and development problems. The scale of works on the application of computer facilities and mathematical modeling to economical investigations and management practice expanded vigorously. The conception of automated control systems appears, designed for the optimization of management for both complex technology and technological processes, and economical systems, such as an enterprise, branch, economical region, and the industry as a whole (on the level of one or several states).

Simultaneously, the understanding of the importance of social-economical factors in the sustainable development problems increased, which resulted in the urge to mathematically correct the description of these factors and their impact on the efficiency of the development process. However, even the most rational solution does not solve all problems, since to implement this solution, corresponding management is necessary. The losses because of poorly organized management can by many times exceed the losses due to less than the best solution: various random factors, which could not be taken into account in advance, can result in considerable deviation of results from the expected ones. More exactly, one must mathematically describe a system of relations between different organizations in the economy of a state or between different states forming an economical union, and have analyzed their role in production management and the ways to increase its efficiency. At present, such investigations are widely employed in examining global problems and the problems of sustainable development on this planet.

In this article, the authors single out a number of global problems which always exerted a great influence on the life of society and discuss the main principles of mathematical modeling as applied to the examination of these problems. As examples, the problems of general circulation of the atmosphere and oceans, biospheric and ecological processes were chosen. At the same time, the major principles and notions of the optimal control theory, which uses mathematical models of the processes under investigation as its components are presented.

Conditionally, the article consists of three parts. In the first part, the approaches of mathematical modeling and mathematical models of the general circulation of the atmosphere and oceans, models of biospheric and ecological processes on this planet are discussed.

In the second part, the main knowledge on the theory of system control and, in particular, on the theory of dynamic control is represented and various applications to the investigation of problems of sustainable development of society are discussed.

In the third part, the authors formulate a number of scientific problems whose solution in the 21st century can exert a significant effect on the development of society as a whole.

2. Mathematical Modeling and the Control Theory in Examining Complex Processes

2.1. Mathematical Modeling and Progress in Science and Technology

The progress in science and technology and, the rapid growth of energetic power of this civilization, produced numerous problems waiting for a deep scientific analysis. First of all, they are produced by the large scale, direct impact of anthropogenic activity on the environment.

In the last decades, it became obvious that the problem of interaction between human civilization and environment is of all-planetary (global) character. The investigations of local character, even being very important, are not sufficient; further development of human civilization is loosely associated with the fate of biosphere as a whole entity.

Conditions for the existence of both humankind and the biosphere, not simply for coexistence, but for mutual evolution and harmonious combination, has become one of the most important scientific and social problems of contemporary life. However, an examination of the conditions for mutual evolution of humankind and the environment does not fit traditional science. First of all, this problem is a multi-dimensional one. It links natural dynamic processes with processes that occur in society. Its final solution means elaborating the conditions for further development of human civilization.

Here, an important role will be played by mathematical models describing dynamic processes in the biosphere with special emphasis on the computer simulation of the processes under study. First of all, this simulation must be an instrument for linking and correlating diverse information, which is inevitable in complex interdisciplinary investigations. Such investigations need a common language. Participants of ecological studies, the physicists, the economists, etc., must understand each other, and the language of mathematical models is such a uniting basis.

2.2. Mathematical Description of Systems and Processes

The development of science is closely associated with construction and use of models that are applied in a widespread investigational method called modeling. Modeling is a way of reflecting reality and originated as early in the antique epoch simultaneously with the recognition of sciene. Nowadays, it is difficult to find a scientific direction, in which the modeling is not employed. In the economic studies and in the development problems, the methods of modeling play the most important role. The methods of modeling are also important in describing biological systems and global processes that occur with the participation of man.

Mathematical modeling of processes of a biological nature is based on the idea of a biological system as an open system, in which the laws of physics and chemistry are valid. Any model must include the conservation laws: the conservation of matter, energy, momentum, etc. This is the first and basic principle. Obviously, this is insufficient for describing biological systems, such as populations, communities, cenoses, and so on. The relationships are also needed that determine the intensity of the fluxes of matter, energy, etc., depending on the state of the individual components of a system.

The conservation laws must be formulated in terms that reflect the most essential features of the processes under consideration. In examining social macrosystems, the conservation laws must be formulated in economic terms. These laws are well known in economics and are called the balance relations. However, such relations are insufficient for constructing models: the values of many quantities (for example, price level, investment structure, etc.) cannot be determined from balance relations. Various processes in society must also be taken into account in these models. However, in describing the processes of a social nature, the closure of a system becomes very complicated. Scientists must forecast the results of their actions and organize the processes of information processing. Thus, together with the description of transformation processes of matter, energy, and motion, a model of information processes must be presented here.

The examination of global (i.e. planetary) processes already has its own history. A start was made by Vernadski, who showed that the evolution of the Earth's biosphere is a result of complex interactions between "inert" and "living" matter. The origin of the atmosphere, structure of the oceans and surface features of land resulted form the life's activity. Vernadski had developed a series of mathematical methods of complex (system) analysis of the processes that occur in the biosphere.

The contemporary state of this problem has a number of peculiarities. Contemporary global processes are a complex superposition of natural evolution of the biosphere and human activity; the intensity of the latter during the last decades became comparable with the intensity of natural processes. Therefore, a model of human activity must be superimposed on the models of climate, matter circulation, and changes in biota.

The methods of mathematical modeling are natural tools for examining global processes of the biosphere. They are, at the same time, an analytical instrument and a language capable of uniting people, who investigate processes that are so diverse. The first investigation stage must contain the descriptions of these processes and the construction of corresponding models.

One of the first attempts to formalize the global description of ecological problems was undertaken by Forrester, who in 1971 proposed a special variant of a model of economic development. This model contained only ecological parameters: the size of the population and the pollution of the environment. The model made it possible to estimate the mutual influence of these parameters and the rate of economic development. In the Forrester model, pollution of the environment was characterized by a dimensionless quantity called the "level of pollution." The model had a hypothetical dependence of this level on the volume of production (or on the volume of capital) and also a hypothetical dependence of the birthrate and mortality on material provision and pollution of the environment. This simple closed model (a system of five equations with five unknowns) allowed the construction of possible development trajectories from prescribed initial conditions. Varying the hypothetical dependencies, one could obtain the whole families of possible trajectories. For the first time, this model demonstrated a possibility ofcombining industrial, social, and ecological processes in a total description.

After Forrester's *The World Dynamics* (1974) and Meadows' *The Limits of Growth* (1975), the ideas for a united planetary development strategy became popular not only among researchers, but also in wide circles of ordinary people. An independent and broad activity field had arisen: the examination of global ecological processes on the basis of mathematical modeling, control theory, and modern information systems. Corresponding investigations are carried out in many countries (USA, England, Russia, Argentina, Japan, Canada etc). The so-called "Rome Club" had appeared.

2.3. Classes of Mathematical Models

The classification of modeling methods can be carried out with respect to different parameters: applications, objects to be modeled, number of details, etc.

The mathematical models can be divided into three associated classes:

- Deterministic models, in the form of equations or inequalities, describing the system's behavior as, for example, differential equations of motion or constrains in a model of costs and production. Such models are called descriptive ones.
- Optimization models, containing an expression to be maximized or minimized under certain conditions. These expressions can be represented in algebraic, or integral, or any other standard form, in which there are algebraic operations, differentiation, and integration. The optimization problems associated with conflicts are treated by a special theory: the theory of games. Since the optimization prescribes the best way of action, such models are called the normative ones.
- Stochastic models are also expressed in the form of equations and inequalities, but having a stochastic sense: for example, they can treat a mean value. The decision-making theory, which is a branch of optimization, deals with the maximization of a mean value of usefulness. Thus, probabilistic expressions and constrains can also be encountered in the framework of optimization.

Different classes of mathematical models and modeling methods can be employed in examining complex processes of human activity and in analyzing the ways of sustainable development. Their various combinations can be used, involving computational and information systems.

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Biographical Sketches

Guri Marchuk is a scientific counselor of the Presidium of the Russian Academy of Sciences. Between 1962–1980 he worked in the Siberian Branch if the USSR Academy of Sciences, first as a director of the Computer Center and then as the Chairman of the Siberian Branch and Vice-President of the USSR Academy of Sciences. Between 1980–1986 he was a Deputy Prime Minister of USSR and the Chairman of the State Committee of Science and Technology. From 1986–1991 he was the President of the USSR Academy of Sciences. Guri Marchuk is an outstanding scholar in the field of numerical and applied mathematics. He has received many honors, including the Fridman, Keldysh, and Karpinski prizes; he is a member of the Academies of Sciences of Bulgaria, Czechoslovakia, Europe, Finland, France, Germany, India, Poland, and Rumania, and an Honorary Professor of Calcutta, Houston, Karlov, Tel-Aviv, Toulouse, and Oregon Universities, and Budapest and Dresden Polytechnic Universities. He is also a member of the editorial boards of many international and several Russian journals, and the Editor-in-Chief of the Russian *Journal of Numerical Analysis and Mathematical Modeling*, published by the Institute of Numerical Mathematics RAS in the Netherlands. Guri Marchuk is the author of a series of monographs on numerical mathematics, numerical simulation of nuclear reactors, numerical methods for the problems in the atmosphere and ocean dynamics, immunology, medicine, and environmental protection. For notable progress in scientific and organizational activities, Guri Marchuk has been awarded with prestigious state honors.

Yuri Osipov has been President of the Russian Academy of Sciences since 1991. Between 1959–1993 he worked in the Ural Branch of the USSR Academy of Sciences. From 1986, he was a director of the Institute of Mathematics and Mechanics, Ural Branch of the USSR Academy of Sciences. In 1992, he was appointed the Chairman of the Commission of the President of Russian Federation on State Prizes in the field of science and technology. He is the Chairman of the Inter-Ministerial Commission on Space. From June 1993, Yuri Osipov has also been the director of the Steklov Institute of Mathematics, Russian Academy of Sciences. Yuri Osipov is an outstanding scholar in control theory, theory of stability, and theory of differential games; he is a member of the American Mathematical Society, and an Honorable Doctor of the Bar-Ilan University, Israel and Santiago University, Chili. He is also a member of the Washington Academy of Sciences and of the World Academy of Arts and Sciences, Washington. Yuri Osipov is the author of many monographs and publications in the field of control theory, theory of stability, and theory of differential games, dynamics, immunology, medicine, and environmental protection. For notable progress in scientific and organizational activities, Yuri Osipov has been awarded with prestigious state honors.